

HW #3 solution set

1) For traditional scaling you scale (decrease) all parameters by a factor of s and the areal density increases by a factor s^2 . So to increase by a factor of 10 you need to scale to by a factor $s = \sqrt{10} = 3.16$.

The follow parameters scale as:

- a) Decreases by s .
- b) Increases by s .
- c) Constant
- d) Decreases by s .
- e) Decreases by s^3 .
- f) Decreases by s .

2) For traditional scaling you scale (decrease) all parameters by a factor of s . The superparamagnetic effect or the magnetic 'trilemma' suggests that the growth of areal density in the future by traditional scaling is limited by enhanced thermal effects in modern recording media. As the volume $V = \pi D^2t/4$ of the grains is reduced in the scaling process, the magnetization of the grains may become unstable due to thermal fluctuations, and data loss may occur. This phenomenon, also referred to as "superparamagnetic effect", became increasingly important in recent years, as new magnetic hard disk drive products are designed for higher areal densities. The weak intergranular exchange coupling allows the longitudinal recording medium to be approximated as a collection of independent particles. The energy barrier for magnetization reversal in the presence of an external magnetic field H , is given by

$$E_B(H, V) = K_u V \left(1 - \frac{H}{H_0}\right)^n$$

where K_u is the magnetic anisotropy density and H_0 is the intrinsic switching field (or coercive field). For simplicity you can simple use $E_B = K_u V$. However depending the data pattern there will be fields generated that make the grain even less stable. that When considering finite temperatures, the energy barrier needs to be compared to the thermal activation energy $k_B T$, where k_B is Boltzmann's constant and T is the absolute temperature. Thermally activated switching is characterized by a time constant τ following the Arrhenius Néel law

$$\tau = \frac{1}{f_0} \exp\left(\frac{E_B}{k_B T}\right)$$

The attempt frequency f_0 is on the order of $10^9 - 10^{12}$ Hz and sets the time scale for thermally activated magnetization reversal. In class we used 10^{10} . This sets a stability requirement of $K_u V$ exceeds $50 k_B T$.

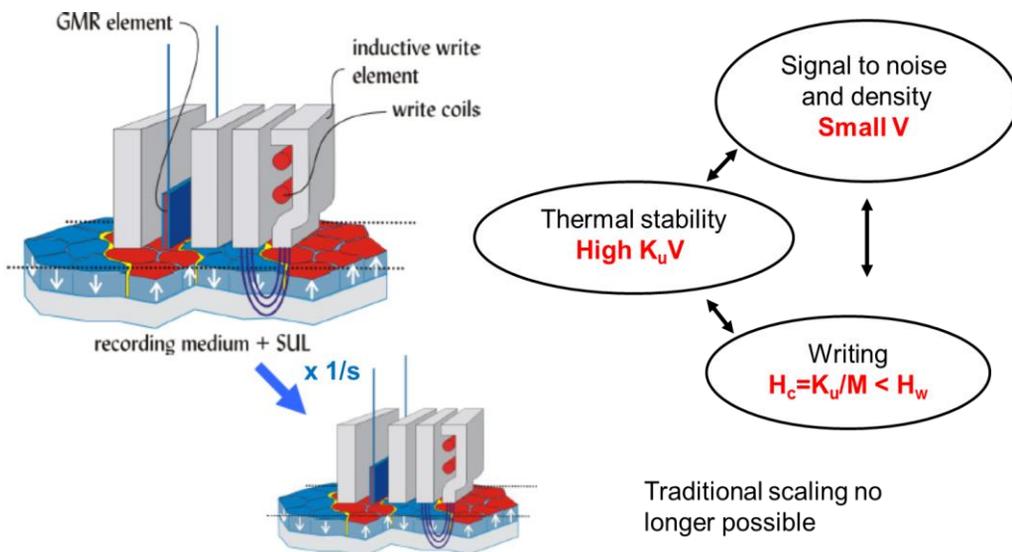
In traditional scaling you need to decrease the grain diameter and thickness with increasing areal density (see problem 1). To first order you want the number of grains per bit to remain constant as you go to higher storage densities and to maintain the read-back signal-to-noise ratio (SNR). Therefore to maintain thermal stability as V decreases as you make the bits (and grains) small you increase K_u .

Nevertheless, following along this path also increases the write field requirements, since the write field

$$H_w \approx H_0 \approx \frac{K_u}{M_s}$$

where M_s is the saturation magnetization of the recording media. The required write field improvements were traditionally achieved by design changes in the write head and the use of materials with higher saturation magnetization as the write poles. However, modern write poles already consist of low-anisotropy materials with saturation magnetization density approaching the highest recorded value. Thus H_w is limited and does not increase with scaling (problem 1).

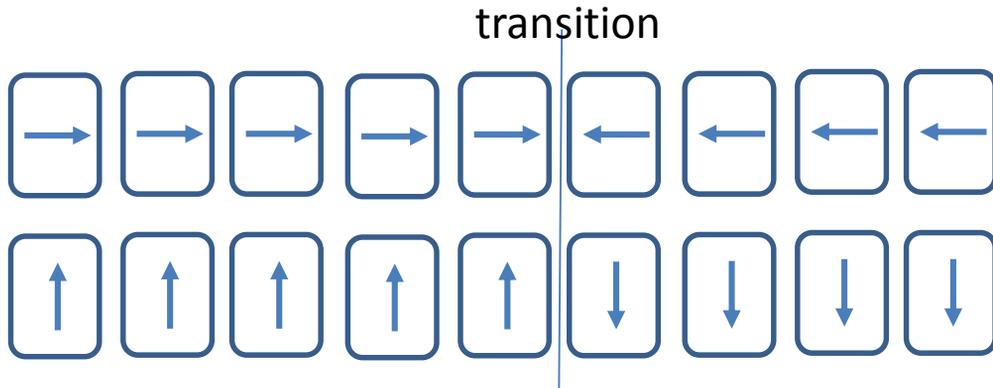
The magnetic trilemma, in short, is that SNR wants small grains (small V), thermal stability requires increased K_u but this is limited by the finite write field from the write head.



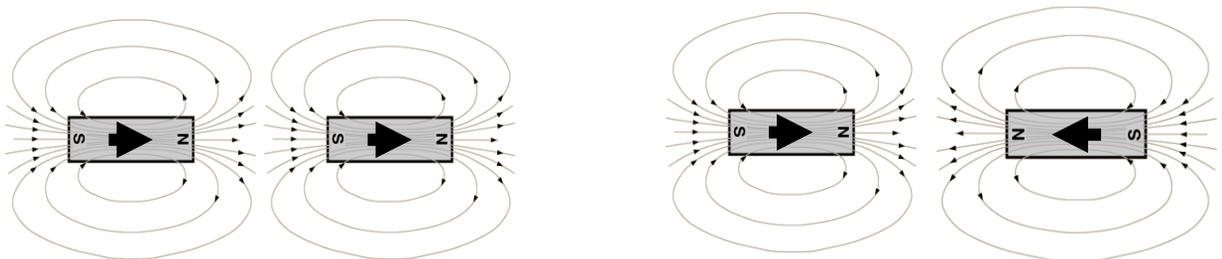
SH Charap, PL Lu, YJ He, IEEE Trans. Magn. **33**, 978-983 (1997) **36 Gbits/in²**

This is a general problem for magnetic information technologies as thermal energies start to dominate the performance when the energy scales become similar to $k_B T$.

3) Below is a schematic of granular media for both longitudinal and perpendicular recording media with a transition (pointing right to pointing left for longitudinal and pointing up to pointing down for perpendicular).



The answer can be seen qualitatively from the two particle interactions. For the longitudinal grains when they are aligned the same direction (left below) the magnetic field from one grain supports the magnetization of the neighboring grains. When they are opposite the field from each grain opposes the neighboring grain lowering the barrier. The two particles on the left are more stable than the two particles on the right so the center of the bit is more stable than the transition.



The case is the opposite for perpendicular grains where dipolar magnetic field from one grain is opposite to the magnetization of the grain next to it. So the particles are more stable when the grain are in the opposite direction (as at the transition) as shown to the right.



To calculate the true stability you need to use:

$$E_B(H, V) = K_u V \left(1 - \frac{H}{H_0} \right)^n$$

where H is the integrated fields from the other grains.

4) If the coercive field is 5 kOe = K/M. Assuming M = 1000 emu/cc yields

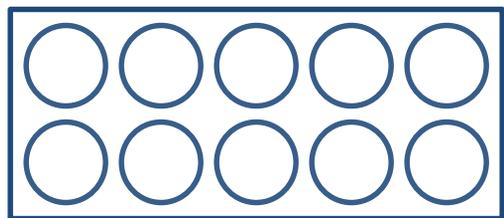
$$K_U = 5 \times 10^6 \text{ ergs/cm}^3.$$

Assuming $K_U V = 75 k_B T$ (I'm using a more stable criterion than the typical 50 $k_B T$ to allow the media to heat and and distributions. At room temperature gives $V = K_U V = 75 k_B T / K_U = 6.21 \times 10^{-19} \text{ cm}^3 = 621 \text{ nm}^3$.

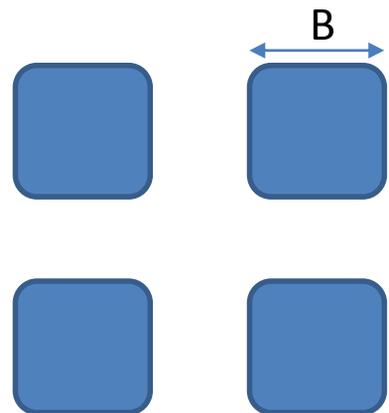
Assuming $V = (\pi D^2/4)t$ and $t=10 \text{ nm}$ then $\pi D^2/4 = 62.1 \text{ nm}^2$ and $D=8.89 \text{ nm}$.

a) Below are 10 grains that have a grain to grain separation of 9.89 nm. So the area that 10 grains takes up is roughly $10 (9.89)^2 = 978.1 \text{ nm}^2$.

Areal density is the # bits/in² so for an inch is 2.54 cm. $\text{in}^2 = 6.45 \text{ cm}^2 = 6.45 \times 10^{14} \text{ nm}^2$.
 Areal density = $6.45 \times 10^{14} \text{ nm}^2 / 978.1 \text{ nm}^2$.
 Equals $6.59 \times 10^{11} \text{ bit/in}^2$ or 659 Gb/in² which is roughly the current drives. Some are approaching 850 Gb/in².



b) For patterned media you have patterned bits that are optimally square with equal size to spacing is shown to the right. For stability $K_U V = K_U B^2 t = 50 k_B T$. Assuming $t=1 \times 10^{-6} \text{ cm}$ then $B = 6.4 \times 10^{-7} \text{ cm} = 6.4 \text{ nm}$. With equal bit to bit spacing one patterned bit takes up $(2B)^2$ area = 165.6 nm².
 Areal density = $6.45 \times 10^{14} \text{ nm}^2 / 165.6 \text{ nm}^2 = 3.89 \times 10^{12} \text{ bit/in}^2$ or 3.89 Tb/in².



c) If you use HAMR and you can now use grains with 40 kOe coercive field. Therefore you can use K_U values 8 times the values in (a) or $K_U = 4 \times 10^7 \text{ ergs/cm}^3$. Therefore the volume of the grains can decrease by a factor of 8. If the thickness has to be 10 nm then the area decrease by 8 or a grain diameter of $8.89 / \sqrt{8} = 3.14 \text{ nm}$.

The grain to grain separation is now 4.14 nm so a bit size of 10 $(4.14)^2 = 171.7 \text{ nm}^2$.

Areal density = $6.45 \times 10^{14} \text{ nm}^2 / 171.7 \text{ nm}^2 = 3.76 \times 10^{12} \text{ bit/in}^2$ or 3.76 Tb/in².

4) The uncertainty in the position of the transition in magnetic recording (known as the jitter) can be estimated by the equation: $\sigma = \frac{\pi^2}{4} a \sqrt{\frac{s}{3W}}$ where a is the width of the transition, s is the cross-track correlation length (the distance over which the magnetic transition fluctuates across the width of the track) and W is the track width. For an uniform, magnetically de-coupled grains the media the transition width is half the grain diameter ($D/2$) and $s = D$.

a) Assuming $W=60$ nm, $D=8$ nm, $s=D=8$ nm and $a=D/2=4$ nm the $\sigma = 2.08$ nm which corresponds to a bit length of 20.8 nm. The area is $BW = 1248.4$ nm².

b) For $W=30$ nm then $\sigma = 2.94$ nm which corresponds to a bit length of 29.4 nm. The area is $BW = 882.8$ nm². The area decreases by sqrt (2) so the areal density increases by sqrt(2) or 41%.

c) It shows that that bits/grain is only a rough estimate since you have higher areal density for squarer bits.

d) Since you get higher areal density for squarer bits for the same grain size the industry has decreased the track width W faster than the bit length B .

5) Assuming you need $50 k_B T$ ($k_B = 1.38 \times 10^{-23}$ J/K). The energy at room temperature is 2×10^{-19} J or 0.2 aJ. The energy to write a flash memory cell is about 1 nJ which is about 5×10^9 times higher energy. This means that there is more than 9 orders of magnitude reduction of energy possible.

6) The critical current for switching is given by:
$$I_C \approx \left(\frac{2e}{\hbar} \right) \frac{2\alpha}{g(\theta)p} E_B$$

Assume $g(\theta) = 1$, $p=0.8$, $\alpha=0.01$ and $E_B= 50 k_B T$ ($k_B = 1.38 \times 10^{-23}$ J/K).
 $e= 1.6 \times 10^{-19}$ C, $\hbar = 1.05 \times 10^{-34}$ J s.

$I_C = 1.6 \times 10^{-5}$ A or 16 μ A. The current density is $I_C/\text{area} = I_C / \pi(15 \text{ nm})^2 = 2.23 \times 10^{-8}$ A/nm² which is 2.23×10^6 A/cm² .