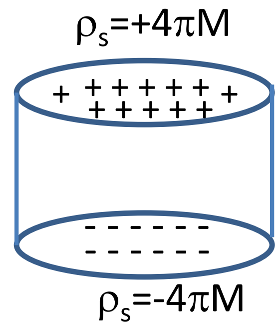
HW #1 solution set

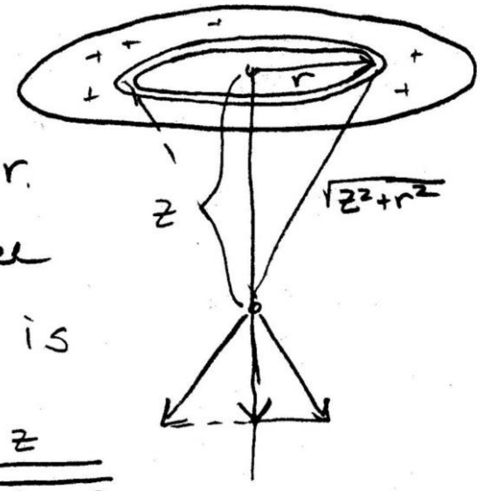
1. a) The charge density at any surface is given by $4\pi\ \hat{n}\cdot\vec{M}$. For the cylinder this means the top has a positive charge density of ρ_s=+4πM and the bottom ρ_s=--4πM.



1 b)

lets calculate the field from a single surface.
Assume a ring of radius r and width dr.
The field at a distance z along the z-axis is given by

$$\frac{2\pi \rho_s r\,dr}{(z^2+r^2)}\ \frac{z}{\sqrt{z^2+r^2}}$$

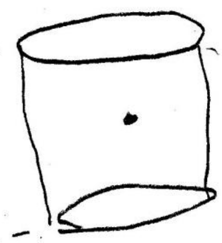


The latter term gives the projection along the z-axis. The total field is

$$H = \int_0^a \frac{2\pi \rho_s z \, r\,dr}{(z^2+r^2)^{3/2}} = \frac{-2\pi \rho_s z}{(z^2+r^2)^{1/2}}\Bigg|_0^a \quad (\rho_s = M)$$

$$= \pm 2\pi M \left(1 - \frac{|z|}{(z^2+a^2)^{1/2}}\right)$$ along the z axis

for 2 surfaces



$$H_z = -2\pi M \left(1 - \frac{|(z-L/2)|}{((z-L/2)^2+a^2)^{1/2}}\right) - 2\pi M \left(1 - \frac{|(z+L/2)|}{((z+\frac{L}{2})^2+a^2)^{1/2}}\right)$$

($-\hat{z}$ direction)

c)

$$H_z = 2\pi M \left(1 - \frac{|(z-L/2)|}{((z-L/2)^2+a^2)^{1/2}}\right) - 2\pi M \left(1 - \frac{|(z+L/2)|}{((z+L/2)^2+a^2)^{1/2}}\right)$$

$+\hat{z}$ direction

d) Inside the magnet $B = H_Z + 4\pi M$ where H_Z is from 2b

Outside the magnet $B = H_Z$ where H_Z is from 1C

e) at $z = L/2$ B_{inside} equals

$$B_{in} = \underbrace{-2\pi M - 2\pi M\left(1 - \frac{L}{(L^2+a^2)^{1/2}}\right)}_{H} + 4\pi M$$

$$B_{in} = 2\pi M \frac{L}{(L^2+a^2)^{1/2}}$$

$$B_{outside} = 2\pi M - 2\pi M\left(1 - \frac{L^2}{(L^2+a^2)^{1/2}}\right)$$

$$= 2\pi M \frac{L}{(L^2+a^2)^{1/2}}$$

@ $z = \frac{L}{2}$ $B_{in} = B_{out}$

f) For $a \rightarrow \infty$ $H_{in} = -4\pi M$ and $B_{in} = H_{in} + 4\pi M = 0$. This is what is expected for a uniform film with the magnetization normal to the surface.

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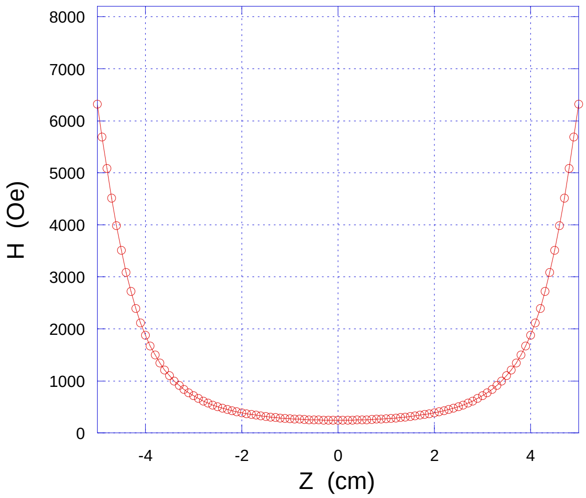
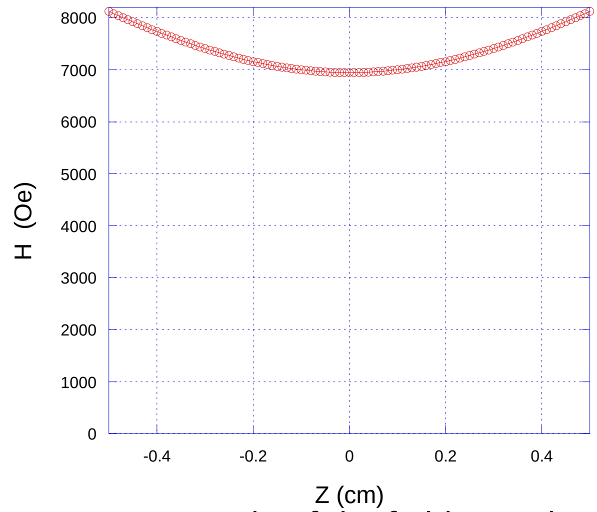
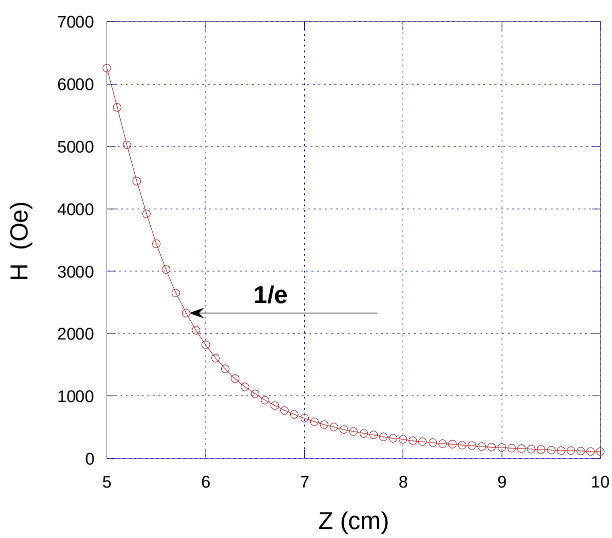
Magnitude of the field inside the L=10 cm magnet



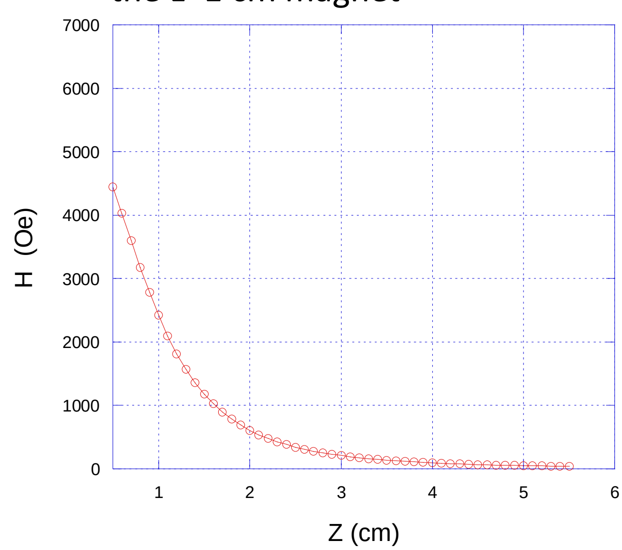
Magnitude of the field inside the L=1 cm magnet



Magnitude of the field outside the L=10 cm magnet



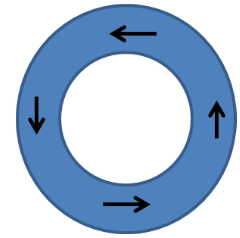
Magnitude of the field outside the L=1 cm magnet



Notice that for the L=10 cm magnetic the magnetic field in the magnet is very inhomogeneous and is much higher near the ends. This often results in the magnet to start to reverse at the ends where there is a large demagnetization field.

1h The 1/e distance (shown above) is about 0.8a so the radius of the magnetic gives the rough length scale over what distance the field will fall off.

2). Magnetic fields are generated by magnetic poles either surface $\rho_s = \hat{n} \cdot \vec{M}$ or bulk $\rho = \nabla \cdot \vec{M}$
Since the magnetization is always parallel to the surface then there are no surface charges. For the bulk charges the divergence of the defined magnetization $\vec{M} = M\hat{\phi}$ (most easily done in cylindrical coordinates) is zero. Since there are no charges there are no fields generated.



3). For a Ni wire where L >> a the internal field within the wire when the magnetization in perpendicular wire is $2\pi M_s$. You need to overcome this field to rotate the magnetization so you need $2\pi M_s$ field which for Ni (M_s = 484 emu/cm^3) yielding an expected saturation field of 3040 Oe.

4). a) assume L = 6cm, a = 0.625cm then the magnetization is M = $7.45\times10^3/(\pi a^2 L)$ = 1000 emu/cc.
b) The magnetic moment of a solenoid is: $m(\text{solenoid}) = \frac{nAi}{10}\frac{\text{erg}}{\text{Oe}}$ = 7.45 x 10^3 emu. For n=200 turns then i= 303.5 A
c) For a typical coil the resistance is about 1 Ω. The power dissipated is I^2R so about 90,000 Watts! This would cause the coil to melt. A superconducting coil has no resistance so can handle large currents. However, even superconducting magnets have their limits (i.e. has a maximum current before they lose their superconducting properties.

5). Assume you can measure 10^{-6} emu signal is typical for a good magnetometer. If you Fe (1700 emu/cm^3) on a 0.1 x 0.1 cm substrate then you can estimate the thickness of Fe to give 10^{-6} emu = 1700 * 0.1 * 0.1 * t. This gives a thickness t=5.9x10^{-8} cm or 0.59 nm. The atomic spacing for Fe is 0.144 nm which is about four atomic layers of Fe.

6 a) For a uniaxial particle: H_C=2K/M.
K=1x10^5 ergs/cm^3 H_C = 200 Oe.
K=1x10^6 ergs/cm^3 H_C = 2000 Oe.
K=1x10^7 ergs/cm^3 H_C = 20,000 Oe.
K=1x10^8 ergs/cm^3 H_C = 200,000 Oe. (this is a huge number!).

b) From the notes $R = \frac{9\sqrt{AK}}{2\pi M^2}$ where A=1x10^{-6} ergs/cm

K=1x10^5 ergs/cm^3 R = 4.53 nm (this is not a physical number since we didn't consider the width of the domain wall.
K=1x10^6 ergs/cm^3 R = 14.3 nm
K=1x10^7 ergs/cm^3 R = 45.3 nm
K=1x10^8 ergs/cm^3 R = 143.2 nm

These number are too small. You need to perform a proper micromagnetic calculation that includes the width of the domain wall and the magnetic fields inside the sphere even in the domain state.

c) The stability for 1 second corresponds to the particle energy KV = 20 k_BT which is the rough estimate for a superparamagnetic particle. k_B = 1.38x10^{-16} erg/K and T=300 K. This leads to R= (15kBT/πK)^1/3.
K=1x10^5 ergs/cm^3 R = 12.6 nm
K=1x10^6 ergs/cm^3 R = 5.8 nm
K=1x10^7 ergs/cm^3 R = 2.7 nm
K=1x10^8 ergs/cm^3 R = 1.3 nm

d) You need to use Sharrock's eq. $$H_C(T) = H_{C0}\left[1-\left(\frac{k_BT}{E_B}\ln\frac{t_p f_0}{\ln 2}\right)^2\right]$$ where H_{c0} is the value from (a) i.e. the zero temperature coercive field, t_p=1 sec, the measurement time and f_0 = 1 x 10^{10} /sec (10 GHz). You know E_B/k_BT at T=300 K is 20. So at 200 K it is now 30 or k_BT/E_B = 1/30. The quantity in the square brackets is 0.39. So at 200 K the HC values measured at 1 sec with 0.39 times the values in (a). At 100 K the k_BT/E_B = 1/60 so the factor in the square bracket is 0.85 so multiply the results in (a) by 0.85.

e) The inverse of the time scale for switching is $\tau^{-1} \sim f_0 \exp(-E_B/k_BT)$ so $t = f_0^{-1} \exp(E_B/k_BT)$. F_0=1x10^{10} and E_B/k_BT equals 30 at 200 K and 60 at 100 K.

At 200 K, τ = 1070 sec. At 100 K, τ = 1.14 x 10^{16} sec or 363,000,000 years!

7. For a superparamagnetic particle with strong anisotropy there will be two states allowed, up and down. This is the same as the quantum calculation of a the Brillion function for J= ½ state. The energy of the two states are: $\exp(mH/k_BT)$ and $\exp(-mH/k_BT)$.

The average magnetization is given by $\langle \mathfrak{m} \rangle = \frac{\sum_i \mathfrak{m}_i \exp(-\varepsilon_i / k_B T)}{\sum_i \exp(-\varepsilon_i / k_B T)}$ which for two states is

$$M = Nm\frac{\exp(mH/k_BT) - \exp(-mH/k_BT)}{\exp(mH/k_BT) + \exp(-mH/k_BT)} = Nm\tanh(mH/k_BT)$$

Which differs from the simple Langevin function for a zero anisotropy particle. The curves are shown below: