Susceptibility $\chi = \frac{dM}{dH}$

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Basic energies

$$E_{external} = -\vec{H} \cdot \vec{M} \qquad E_{stray} = -\frac{1}{2}\vec{H}_{demag} \cdot \vec{M}$$

$$E_{exchange} = A\left(\frac{\partial \theta}{\partial x}\right)^2 \qquad E_{anisotropy} = g(\theta)$$

$$= K_U \sin\theta^2$$

For uniaxial anisotropy

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$$\langle \mathrm{m} \rangle = \frac{\sum_i \mathrm{m}_i \exp(-\varepsilon_i / k_B T)}{\sum_i \exp(-\varepsilon_i / k_B T)}$$

Boltzmann average

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Domain wall: Energy and Width

$$\sigma_w = 4\sqrt{AK} \quad \textit{and} \quad \delta_w = \pi\sqrt{A/K}$$

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Exchange length: Strong K

$$l \approx \pi\sqrt{A/K}$$

shape dominated

$$l \approx \pi\sqrt{A/2\pi M^2} \approx \sqrt{A}/M$$

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Demagnetization energy

$$\vec{H}_d(\vec{r}) = -\tilde{N}(\vec{r}) \cdot \vec{M} \qquad \sum_{i=1}^{3} N_i = 4\pi$$

$N_x = N_y = 0$ , $N_z = 4\pi$ for film

$N_x = 0$ , $N_y = N_z = 2\pi$ for cylinder

$N_x = N_y = N_z = 4\pi/3$ for sphere

## Thermal energy

$$E_B \sim K_U V(1-H/H_0)^n$$

$$\tau^{-1} \sim f_0 \exp(-E_B/k_B T)$$

$$H_C(T) = H_{C0}\left[1-\left(\frac{k_B T}{E_B}\ln\left(\frac{t_P f_0}{\ln 2}\right)\right)^n\right]$$

## Dynamics

$$\frac{d\mathbf{m}}{dt} = \gamma\, \mathbf{H}\times\mathbf{m} + \alpha\left(\mathbf{m}\times\frac{d\mathbf{m}}{dt}\right)$$

$$\vec{H}_{eff} = -\partial E_{tot} / \partial \vec{M}$$

$$\gamma_e = \frac{|-e|}{2m_e} g_e = g_e \mu_B / \hbar,$$

Larmor frequency $\omega = 2\pi f = \gamma \boldsymbol{H}_{eff}$

$\gamma/2\pi$ = 28 GHz / Tesla

For protons $\gamma/2\pi$ = 42.58 MHz / Tesla

## Thin films

$$K = K_{eff} = K_v + 2K_s/t.$$

$K_V$ = volume anisotropy

$K_S$ = surface anisotropy

$$\frac{E}{A} = -J\, \hat{m}_1 \cdot \hat{m}_2$$

Where J is the coupling constant

$$\frac{\Delta R}{R_{avg}} = \frac{R^{\parallel} - R^{\perp}}{(R^{\parallel} + 2R^{\perp})/3}$$

Anisotropic magneto-resistance

$$MR = \frac{R_{antiparallel} - R_{parallel}}{R_{parallel}}$$

For GMR and TMR

$$\Delta R = -(1/2)(R_\uparrow - R_\downarrow)^2 / (R_\uparrow + R_\downarrow)$$

Two-current channel model of GMR

$$MR = \frac{R_{AP} - R_P}{R_P} = \frac{2P_1 P_2}{1 - P_1 P_2}$$

$$\text{with } P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

Juliere model for tunneling